ECON/AMS 11A — Midterm Exam — Solutions — FALL 2017

November 3, 2017

(1) The savings function for a small country is given by

$$S = \frac{Y^2 + 2Y + 1}{9Y + 20},$$

where Y is annual income and S is annual savings, both measured in billions of dollars per year.

a. (5 pts) Find the nation's *marginal propensity to save*, $\frac{dS}{dY}$, and *marginal propensity to consume*, $\frac{dC}{dY}$, when national income is \$10 billion.

$$\frac{dS}{dY} = \frac{(2Y+2)(9Y+20) - 9(Y^2+2Y+1)}{(9Y+20)^2} = \frac{9Y^2 + 40Y + 31}{(9Y+20)^2}$$

so

$$\left.\frac{dS}{dY}\right|_{Y=10} = \left.\frac{9Y^2 + 40Y + 31}{(9Y+20)^2}\right|_{Y=10} = \frac{1331}{12100} = 0.11$$

and

$$\left.\frac{dC}{dY}\right|_{Y=10} = 1 - \left.\frac{dS}{dY}\right|_{Y=10} = 0.89.$$

b. (3 pts) Find $\lim_{Y\to\infty} \frac{dS}{dY}$ and interpret your answer in economic terms.

$$\lim_{Y\to\infty} \frac{dS}{dY} = \lim_{Y\to\infty} \frac{9Y^2 + 40Y + 31}{(9Y+20)^2} = \lim_{Y\to\infty} \frac{9Y^2 + 40Y + 31}{81Y^2 + 360Y + 400} = \lim_{Y\to\infty} \frac{9Y^2}{81Y^2} = \frac{1}{9}.$$

Interpretation: *When income is large, the nation will tend to save about* $1/9$ *of every additional dollar of income.*

(2) The ***demand equation*** for a monopolistic firm's product is

$$q = 100 - 0.25p^2,$$

where p is the price per unit (in dollars) of the firm's product and q is the weekly demand for the firm's product, measured in 100s of units.

a. (3 pts) Find the *price-elasticity of demand* for this firm's product when the price is $p_0 = \$10$. ***Show your work.***

$$\eta_{q/p} = \frac{dq}{dp}\cdot\frac{p}{q} = \overbrace{-0.5p}^{dq/dp}\cdot\frac{p}{100 - 0.25p^2} = -\frac{0.5p^2}{100 - 0.25p^2},$$

so

$$\left.\eta_{q/p}\right|_{p=10} = -\left.\frac{0.5p^2}{100 - 0.25p^2}\right|_{p=10} = -\frac{50}{75} = -\frac{2}{3}.$$

b. (2 pts) Use your answer to **a.** to find the firm's ***marginal revenue*** dr/dq when the price is $p_0 = \$10$.

$$\left.\frac{dr}{dq}\right|_{p=10} = p\left(1 + \frac{1}{\eta_{q/p}}\right)\bigg|_{p=10} = 10 \cdot \left(1 + \frac{1}{-2/3}\right) = -5.$$

c. (3 pt) What will be the approximate ***percentage change*** in the demand for the firm's product if they raise the price of their good from $p_0 = \$10$ to $p_1 = \$10.25$? *Explain your reasoning.*

Linear approximation for percentage change:

$$\%\Delta q \approx \eta_{q/p} \cdot \%\Delta p = -\frac{2}{3} \cdot \overbrace{\left(\frac{0.25}{10} \cdot 100\%\right)}^{\%\Delta p} = -\frac{2}{3} \cdot 2.5\% \approx -1.667\%$$

(3) The ***marginal revenue*** function for ACME Widgets Inc. is

$$\frac{dr}{dq} = \frac{10q}{0.07q^2 + 13},$$

where q is monthly output, measured in 100s of widgets, and r is monthly revenue, measured in \$1000s.

ACME's production function is

$$q = 5(3l + 4)^{1/2},$$

where l is the number of FTEs that the firm employs per month.

An FTE is a *full-time equivalent*: a person working a 40-hour work week counts as 1 FTE, while a part-time laborer who works 30 hours a week, for example, counts as 0.75 FTEs.

a. (3 pts) Compute ACME's ***output*** and ***marginal product of labor***, dq/dl, when $l_0 = 20$.

$$\left.q\right|_{l=20} = 5 \cdot 64^{1/2} = 40 \qquad \text{and} \qquad \left.\frac{dq}{dl}\right|_{l=20} = \frac{15}{2}(3l+4)^{1/2}\bigg|_{l=20} = \frac{15}{16}.$$

b. (2 pts) Compute ACME's ***marginal revenue product***, dr/dl, when $l_0 = 20$.

$$\overbrace{\left.\frac{dr}{dl}\right|_{l=20} = \left.\frac{dr}{dq}\right|_{q=40} \cdot \left.\frac{dq}{dl}\right|_{l=20}}^{\text{chain rule}} = \left.\frac{10q}{0.07q^2+13}\right|_{q=40} \cdot \frac{15}{16} = \frac{400}{125} \cdot \frac{15}{16} = 3$$

c. (3 pts) Suppose that ACME hires a new part-time widget washer to work 10 hours a week. By *approximately* how much will ACME's ***monthly revenue*** change? Express your answer in dollars.

Linear approximation:

$$\Delta r \approx \left.\frac{dr}{dl}\right|_{l=20} \cdot \Delta l = 3 \cdot 0.25 = 0.75$$

so monthly revenue will increase by about $0.75 \cdot \$1000 = \$750$.

Remember — l is measured in 40-hour work weeks, so an increase of 10 hours/week in labor input means that $\Delta l = 10/40 = 0.25$.

(4) Find the derivatives of the following functions. ***Clean up your answers.***

a. (3 pts) $y = \dfrac{3x^2 + x + 1}{5x - 2}$

$$\frac{dy}{dx} = \frac{(6x+1)(5x-2) - 5(3x^2+x+1)}{(5x-2)^2} = \frac{15x^2 - 12x - 7}{(5x-2)^2}$$

b. (2 pts) $g(t) = 3\ln(t^4 + 2t - 2)$

$$g'(t) = 3 \cdot \frac{4t^3 + 2}{t^4 + 2t - 2} = \frac{12t^3 + 6}{t^4 + 2t - 2}$$

c. (3 pts) $f(u) = 5e^u \cdot u^3$

$$f'(u) = 5e^u \cdot u^3 + 5e^u \cdot 3u^2 = 5e^u(u^3 + 3u^2)$$